CBCS SCHEME

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15MAT31

Third Semester B.E. Degree Examination, June/July 2023
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Obtain the Fourier series for $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$. **(08 Marks)**

b. The following table gives the variations of a periodic current over a period.

t(sec)	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (amp)	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a direct current part of 0.75 amps in the variable current and obtain the amplitude of the first harmonics. **(08 Marks)**

OR

2 a. Obtain the Fourier series for the function :

$$f(x) = 1 + \frac{2x}{\pi} \text{ in } -\pi < x < 0$$

$$1 - \frac{2x}{\pi} \text{ in } 0 < x < \pi .$$ **(04 Marks)**

b. Obtain the half-range sine series for the function :

$$f(x) = \begin{cases} x & \text{for } 0 < x < \pi/2 \\ \pi - x & \text{for } \pi/2 < x < \pi. \end{cases}$$ **(06 Marks)**

c. Express y as a Fourier series upto the second harmonics given,

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/6$	2π
y	4	8	15	7	6	2	4

(06 Marks)

Module-2

3 a. Find the Fourier transform of the function :

$$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$

Hence evaluate : $\int_0^\infty \frac{\sin ax}{x} dx$. **(06 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$, show that $\int_0^\infty \frac{x \sin mx}{1+m^2} dx = \frac{\pi}{2} e^{-m}$. **(05 Marks)**

c. Find the Z-transform of sin (3n + 5). **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

4 a. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$

Hence evaluate : $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cdot \cos\frac{x}{2}\, dx$. **(06 Marks)**

b. Find the z-transform of $\cosh n\theta$. **(04 Marks)**

c. Solve the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ given $y(0) = 0$, $y(1) = 1$. **(06 Marks)**

Module-3

5 a. Fit a second degree parabola in the form $y = a + bx + cx^2$ to the following data :

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

(06 Marks)

b. If θ is the angle between the two regression lines show that :

$$\tan\theta = \frac{1-r^2}{r}\frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma y^2}$$

Explain the significance when $r = 0$ and $r = \pm 1$. **(05 Marks)**

c. Compute the real root of $x\log_{10}x - 1.2 = 0$ by the method of false position. Carry out 3 iterations. **(05 Marks)**

OR

6 a. If P is the pull required to lift a load W by means of a pulley block, find a linear law of the form $P = m\,w + c$ connecting P and W, using the data :

P	12	15	21	25
W	50	70	100	120

(05 Marks)

b. Obtain the lines of regression and hence find the coefficient of correlation for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(06 Marks)

c. Use Newton – Raphson method to find a real root of $x\sin x + \cos x = 0$ near $x = \pi$. Carry out three iterations. **(05 Marks)**

Module-4

7 a. The following data gives the values of tan x for $0.10 \le x \le 0.30$. Find tan (0.26) by using Newton's backward formula :

x	0.10	0.15	0.20	0.25	0.30
tan x	0.1003	0.1511	0.2027	0.2553	0.3093

(06 Marks)

b. Use Lagranges interpolation formula to find y at x = 10 given :

x	5	6	9	11
y	12	13	14	16

(05 Marks)

c. Use Simpson's $\frac{1}{3}^{rd}$ rule with 7 ordinates to evaluate : $\int_2^8 \frac{dx}{\log_{10} x}$. **(05 Marks)**

OR

8 a. Given f(40) = 184, f(50) = 204, f(60) = 226, f(70) = 250, f(80) = 276, f(90) = 304, find f(85) using Newton's backward interpolation formula. **(05 Marks)**

b. Find f(2) using Newton's divided difference formula given the values :

x	0	1	4	8	10
f(x)	–5	–14	–125	–21	355

(06 Marks)

c. Evaluate $\int_0^1 \frac{x\,dx}{1+x^2}$ by Weddle's rule taking seven ordinates. **(05 Marks)**

Module-5

9 a. Verify Green's theorem for $\int_C (xy+y^2)dx + x^2 dy$ where C is the bounded by y = x and $y = x^2$. **(06 Marks)**

b. Verify Stoke's theorem for $\vec{F} = (2x-y)i - yz^2 j - y^2 zk$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$, c is its boundary. **(05 Marks)**

c. Find the extremal of the functional $I = \int_0^{\pi/2} (y^2 - y'^2 - 2y\sin x)dx$ under the end conditions :

$y(0) = y\left(\frac{\pi}{2}\right) = 0$. **(05 Marks)**

OR

10 a. If $\vec{F} = 2xyz' + yz^2 j + xzk$ and s is the rectangular parallelepiped bounded by x = 0, y = 0, z = 0 x = 2, y = 1, z = 3. Evaluate $\iiint_V \text{div}\bar{f}dv$. **(06 Marks)**

b. Solve the variational problem :

$\delta \int_0^{\pi/2} (y^2 - y'^2)dx = 0$; y(0) = 0 y(π/2) = 2. **(05 Marks)**

c. Find the geodesics on a surface given that the arc length on the surface is

$S = \int_{x_1}^{x_2} \sqrt{x(1+y'^2)}dx$. **(05 Marks)**

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